Transonic Shockwave/Turbulent-Boundary-Layer Interaction with Suction or Blowing

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An approximate nonasymptotic theory of weak normal shock-unseparated two-dimensional turbulent boundary-layer interactions is given which includes the effect of surface mass transfer. Mainly as a consequence of its influence on the boundary-layer profile shape away from the wall, it is shown that even small amounts of suction ($-\dot{m}_w/\rho_e u_e \lesssim 5\times10^{-4}$) appreciably reduce the upstream influence and thickening effects of the interaction but hasten the onset of separation. Detailed results of a parametric study of typical suction and blowing effects on interactive pressure, thickness, and skin friction distributions are also presented.

Nomenclature

B = mass transfer parameter $= \dot{m}_w/\rho_e u_e$
C_f = skin-friction coefficient $= 2\tau_w/\rho_e u_e^2$
L = distance to undisturbed shock location
$\dot{m}_w$ = mass flux rate (per second) across surface $= \rho_{w_0} v_{w_0}$
M = Mach number
ΔP = pressure rise across incident normal shock
p' = interaction pressure perturbation $= p - p_1$
Re_L, Re_δ = Reynolds numbers $= \rho_e u_e L/\mu_e, \rho_e u_e \delta/\mu_e$
T = absolute static temperature
u,v = flow velocity components along x,y, respectively
x,y = streamwise and normal coordinates, respectively
γ = ratio of specific heats
δ = undisturbed boundary-layer thickness
δ^* = displacement thickness
η $= y/\delta_0$
$\Delta\eta$ = interactive boundary-layer thickening (see Fig. 1)
μ = coefficient of viscosity
ν = kinematic viscosity coefficient, μ/ρ
ρ = density
τ = shear stress
θ^* = momentum thickness

Subscripts

1,2,3 = interaction regions (see Fig. 1)
e = edge of boundary layer
0 = undisturbed solid-wall boundary-layer conditions
ref = Eckert reference temperature value
w = wall surface conditions

Introduction

THE study of transonic interactions between shockwaves and turbulent boundary layers is important in the aerodynamic design of high-speed aircraft wings, turbine and cascade blades in turbomachinery, and airbreathing engine inlets and diffusors. Consequently, the control and suppression of interaction effects in these applications by suction or blowing is of great interest, since boundary-layer control (BLC) by surface mass transfer has advanced to a very practical status. Experimental studies have established the practical value of using distributed suction normal to the surface for shock/boundary-layer interaction separation suppression on wings,[1,2] porous wind tunnel walls,[3] and within supersonic inlets.[4] There is also interest in normal *blowing* effects in connection with transonic shock/boundary-layer interactions on wings[5] and on mass-transfer-cooled hot turbine blades.[6] Moreover, the study of interactions in the presence of distributed normal mass transfer is of fundamental interest in its own right[7] and provides an idealized model for optimum performance estimates of BLC systems on wings and flaps and rational criteria for separation prevention by suction.[8]

Although a modest body of basic theory for the transonic shock/turbulent-boundary-layer interaction problem has accumulated for the zero mass transfer case,[9-11] only rather crude overall integral methods are presently available to treat the effects of suction or blowing[4]; a more detailed basic theory including mass transfer is thus desirable to provide a proper analytical framework and understanding in the aforementioned applications. Accordingly, the present paper describes such a theory for the restricted but important case of nonseparating flow with normal wall (unvectored) mass transfer. It is based on an extension of the analytical theory of weak normal shock/turbulent-boundary-layer interactions developed by Inger and Mason.[11]

Outline of Theory

For unseparated turbulent interacting boundary layers ($M_1 \lesssim 1.3$) it is possible to construct a basic theory of the problem wherein the flow consists of a known incoming isobaric turbulent boundary-layer profile $M_0(y)$ subjected to small transonic disturbances due to an impinging weak normal shock. In the practical Reynolds number range of interest here ($10^5 \le Re_L \le 10^8$), we employ a nonasymptotic disturbance flow model in the turbulent boundary layer patterned after the Lighthill-Stratford-Townsend double-deck approach[12,13] that has proven highly successful in treating a variety of other problems involving turbulent boundary-layer

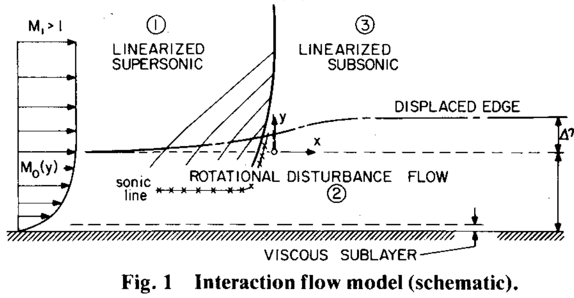

Fig. 1 Interaction flow model (schematic).

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response to strong rapid adverse pressure gradients. This was done because of the large body of turbulent-boundary-layer/shock interaction data that strongly supports such a model in this Reynolds number range[4,8] (including the present transonic regime), and the findings of a separate general theoretical study[14] suggesting that the results of the asymptotic theory approach for very high Reynolds numbers,[9,10] although rigorous in this limit, may not extrapolate down to the present Reynolds number range.

The resulting flow model consists of an inviscid boundary value problem surrounding a *nonlinear* shock discontinuity and underlaid by a thin viscous disturbance sublayer as schematically illustrated in Fig. 1. An approximate analytical solution is further achieved in the leading approximation by introducing some engineering simplifications[11] (including the assumption of small linearized disturbances ahead of and behind the *nonlinear* shock jump plus neglect of the detailed shock structure within the boundary layer[15]) which have been shown to give accurate results for the overall properties of engineering interest, provided M_1 is not too close to unity ($M_1 \gtrsim 1.05$‡). Thus, the supersonic disturbance flow in region 1 is governed by Ackeret's classical linearized theory with incoming waves ruled out, while the subsonic flow in region 3 (caused by the interaction-generated interface displacement $\eta_3(x)$ plus the post-shock perturbations along $x=0^+$ resulting from the impingement of region 1 Mach wave disturbances on the shock) is governed by a linearized elliptic boundary value problem that can be solved by a Fourier sine transformation approach. The rotational inviscid disturbance flow region 2 within the boundary layer profile is treated by Fourier transformation with respect to x in a manner similar to one used earlier by Lighthill in a related problem,[17] a suitable model of $M_0(y)$ being a required input to this solution. Finally, within the thin viscous disturbance sublayer (which carries most of the upstream influence of the interaction), a known solution[17] for a linear basic profile $u_0=(\tau_w/\mu_w)y$ can be carried over to the present problem to obtain the sublayer displacement effect (effective wall position seen by the overlying inviscid disturbance flow) and also the corresponding disturbance skin friction caused by the pressure disturbance field.

The matching of the aforementioned regional solutions yields readily solved linear integral equations for the disturbance pressure along both the boundary-layer edge and wall.[11] The remaining interactive flow properties can then be determined in terms of p_w'. For example, double integration of the combined continuity and y-momentum equations across the width of the inviscid rotational disturbance region in the boundary layer yields the local increase in displacement thickness. The viscous disturbance-flow solution in the underlying viscous sublayer determines the interactive skin-friction perturbation associated with p_w'. Upon extending this solution to include downstream as well as upstream of the shock and further correcting it for the important nonlinear inertia effects in an adverse pressure gradient using the general nondimensional wall shear-pressure solution ahead of separation by the triple deck theory[18] (converted to turbulent flow by expressing all results in terms of C_{f_0} instead of Re_L), it is found that

$$C_f(x) \simeq C_{f_0} - \sqrt{\beta C_{f_0}\left[1+\left(\frac{C_f(x)-C_{f_0}}{1.234\, C_{f_0}}\right)\right]}\, C'_{p_w}\cdot F\left(\frac{x}{\delta_0}\right) \quad (1)$$

where the nondimensional function F is essentially unity ahead of the shock $x<0$ and vanishes slowly behind it with $F \sim (x/\delta_0)^{-1/3}$ far downstream. It is seen from Eq. (1) that depending on Reynolds number (C_{f_0}), a sufficiently strong interactive pressure rise can cause incipient separation ($C_f \to 0$) near the shock.

Detailed comparisons[19] have shown that the results give a good account of the important engineering aspects of the interaction (e.g., $\partial p/\partial y$ effects across the boundary layer near the shock, interaction pressure rise along the wall, displacement thickness growth, and interactive skin friction) over a wide range of Mach-Reynolds number conditions at very low computational cost. Hence the theory provides a sound basis for interpreting experimental data on unseparated flows and for further extensions. In particular, the theory can be adapted to allow mass transfer through the surface, and in the following we briefly highlight how this was done (full details are given in Ref. 20).

Surface mass transfer influences the interaction in two ways: 1) it alters the incoming undisturbed flow (on which the disturbance solution depends) by changing τ_{w_0} and δ_0, the damping of turbulence across the laminar sublayer, and the profile shape away from the wall; 2) it further introduces new mass transfer-induced terms in the governing disturbance equations. Now the first effects are by far the most important and will be treated separately below. The secondary effects may, in fact, be neglected with good approximation under the assumed conditions of small-to-moderate normal mass transfer rates ($|\dot{m}_w/\rho_{e_1}u_{e_1}| \lesssim 10^{-3}$) typical of practical applications, according to the following considerations. Under the continued assumption that the viscous disturbance sublayer lies within the laminar sublayer region of a turbulent boundary layer, the mass transfer effect in the leading approximation does not introduce any curvature into the $u_0(y)$ profile but only alters its *slope* (τ_w); consequently in the viscous disturbance sublayer perturbation equations those terms proportional to d^2u_0/dy^2 and v_{w_0} (which in turn is proportional to d^2u_0/dy^2 via the mean flow momentum equation near the wall) can be neglected, leaving the *form* of these equations unchanged. Likewise, under the continued assumption that turbulent fluctuations are uncorrelated with the interactive disturbances, the explicit new terms in the overlying rotational-inviscid perturbation equations that are proportional to v_{w_0} can be neglected also, since detailed studies of the hydrodynamic stability equations (which are very similar to those of the present problem) have shown[21] that these terms have an altogether negligible effect throughout a high Reynolds number, parallel shear flow boundary layer unless the surface mass transfer is quite large. Thus to a consistent degree of first approximation, the form of both the viscous and inviscid disturbance equations is unchanged by moderate blowing or suction provided their effect on the mean flow-based coefficients in these equations *is* included. It is reemphasized that the primary mass transfer effects on the viscous sublayer field and its thickness are thus fully accounted for.

The present interactive perturbation solution in principle may be used with any mean turbulent boundary-layer profile input and hence could be coupled with either an experimental measurement or any desired state-of-the-art numerical prediction code. Here, to bring out clearly and efficiently where the various mass transfer effects enter, we have chosen an approximate but accurate analytical profile model that has proven especially well suited to such nonuniform flow perturbation problems.[22] We assume for simplicity that the blowing or suction is on the average uniform and normal to the wall, that its streamwise extent is large compared to the short interaction range, and that it extends far enough upstream to have established a well-defined local equilibrium profile in the incoming boundary layer. Then the mass transfer effect on turbulent skin friction can be described by the relation[23]

$$\frac{C_f}{C_{f_0}} \simeq 1 - 23\left(\frac{T_{\rm ref}}{T_e}\,\frac{1}{2C_{f_0}}\right)^{1/2}\cdot B \quad (2)$$

‡As far as the overall interaction solution for $10^5 \leq Re_L \leq 10^8$ is concerned, these nonlinear shock jump conditions plus the various nonuniform viscous flow effects within the boundary layer reduce the lower Mach number limit otherwise pertaining to the linearized supersonic theory in purely inviscid potential uniform flow. (See Ref. 16 for a more detail discussion.)

where

$$T_{ref}/T_e \simeq 1 + 0.038M_e^2 + 0.50(T_w/T_e - 1) \quad \text{for} \quad \gamma = 1.4$$

and C_{f_0} is the zero-blowing value. (We used a reference temperature-based Schulz-Grünow relation.[7]) According to Eq. (2), suction, for example, increases τ_w, and this should have an influence on the interaction that is qualitatively similar to decreasing the Reynolds number. The associated mass transfer effect on boundary-layer thickness is estimated assuming a Crocco energy integral and power-law-type profile relationship between δ and θ^*, with the momentum thickness θ^* found by an approximate integration[20] of the momentum integral equation including mass transfer. In addition to τ_w there is also a mass transfer effect on the profile shape away from the wall as, for example, incorporated in the following turbulent boundary-layer shear stress profile relation recommended by Conrad and Donaldson[24] (similar expressions have been proposed by other investigators, see, e.g., Ref. 25):

$$\frac{\tau}{\tau_w} \simeq 1 - 3\eta^2 + 3\eta^3 + (1-\eta^2)\left(\frac{u}{u_e}\right)\left(\frac{2B}{C_{f_0}}\right) \tag{3}$$

where $\eta \equiv y/\delta$. Further, Eq. (3) is to be used with the basic turbulent shear stress definition that

$$\frac{d(u/u_e)}{d\eta} = \frac{C_f}{2} \cdot Re_\delta \left(\frac{T}{T_e}\right)\left(\frac{\tau/\tau_w}{\nu_{eff}/\nu_e}\right) \tag{4}$$

Regarding the turbulent kinematic eddy viscosity distribution ν_{eff}, the available experimental evidence[25,26] (including some very recent detailed measurements[27]) implies that its functional form is significantly affected only by relatively large surface mass transfer rates provided the mass transfer effect on the value of τ_w *is* taken into account. Thus the existing $B=0$ two-layer piecewise-continuous viscosity formulation developed for use in interaction problems by Inger and Williams[22] may be applied also to the weak-to-moderate blowing or suction cases studied here (indeed this is fully consistent with the aforementioned simplifications). Then Eqs. (2-4) plus the Crocco integral temperature profile $T(u)$ and subsequent integral yield an accurate yet fundamentally based determination of the incoming turbulent boundary-layer velocity and Mach number profiles including compressibility, heat transfer, and moderate amounts of wall suction or injection. The results satisfy the proper boundary conditions, including vanishing gradients at the boundary-layer edge; conform to the law of the wall near the surface; are continuous across the entire boundary layer with a velocity defect-type behavior in the outer part; and have been shown to be in good agreement with experiment over a wide range of transonic to moderately supersonic Mach numbers,[22] including the effects of surface mass transfer.[25]

Discussion of Results

Figure 2a illustrates the typical mass transfer effect on the Mach number profile, showing how this is dominated by the influence on the profile shape away from the wall; including only the τ_w effect gives a significant error in both magnitude and sign of the profile changes. This may be verified from Eq. (2)—inserting typical values in the right-hand side showed that a given amount of blowing (for example) *increases* the local shear stress outside the law of the wall region far more than it reduces τ_w. The typical consequences of this on the interaction solution itself are illustrated in Fig. 2b, which shows how the various contributions to the suction/blowing effect influence the wall pressure distribution (analogous results are obtained for the interactive displacement thickness and skin friction[20]). Whereas the contribution of the mass transfer effect on δ is negligible compared to that on τ_w, the

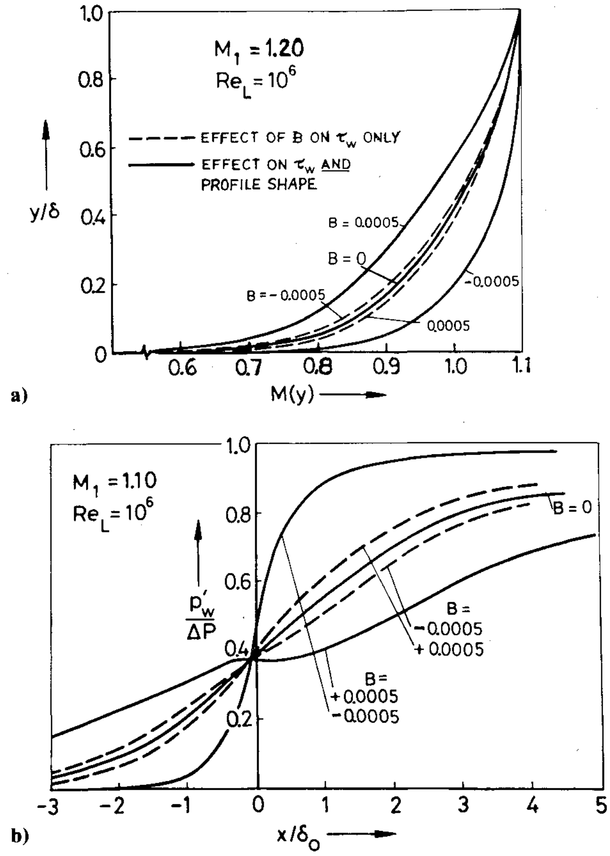

Fig. 2 Typical suction and blowing influence on incoming Mach number: a) profile, b) wall interaction pressure.

influence on profile shape is quite large (in fact completely opposite to and overwhelming the τ_w effect) as indeed would be expected from the aforementioned influence on $M_0(y)$. This conclusion, which was found to apply over a wide range of conditions, is concordant with the finding of Panaras and Inger[28] that transonic interactions are also very sensitive to changes in the turbulent boundary-layer profile form factor due to upstream pressure gradients in the incoming flow.

Referring hereafter to the complete mass transfer model, we observe that suction, because of its predominant effect in decreasing the Mach number gradient and hence enhancing the profile "fullness" away from the wall, reduces the streamwise extent and thickening of the interaction, making it appear more inviscidlike in character with a steeper adverse pressure gradient. Thus suction is qualitatively equivalent to an increase in Reynolds number. Blowing has the expected opposite effects of spreading out the interaction pressure field and increasing the displacement thickness.

Results of a systematic study of suction/blowing effects in terms of the basic mass transfer parameter B for a typical interaction case are illustrated in Fig. 3. In Fig. 3a it is seen that even moderate amounts of suction significantly reduce the upstream influence distance and overall streamwise extent of the interaction and steepen the adverse wall pressure gradient, whereas blowing has equally the opposite effect. Concordant with these trends, suction also strengthens the local shock jump at the boundary-layer edge while reducing (perhaps even eliminating at high enough B) the degree and extent of the postshock expansion region (Fig. 3b). The corresponding interaction-induced growth of the boundary-layer displacement thickness is of great practical interest since this often has a significant back-effect on the inviscid flow and shock position on an airfoil or in channel flows.[19] As shown in Fig. 4 this thickening is strongly influenced by mass transfer; for example, the moderate suction value $B = -0.0003$ reduces $\Delta\eta(\infty)$ nearly threefold.

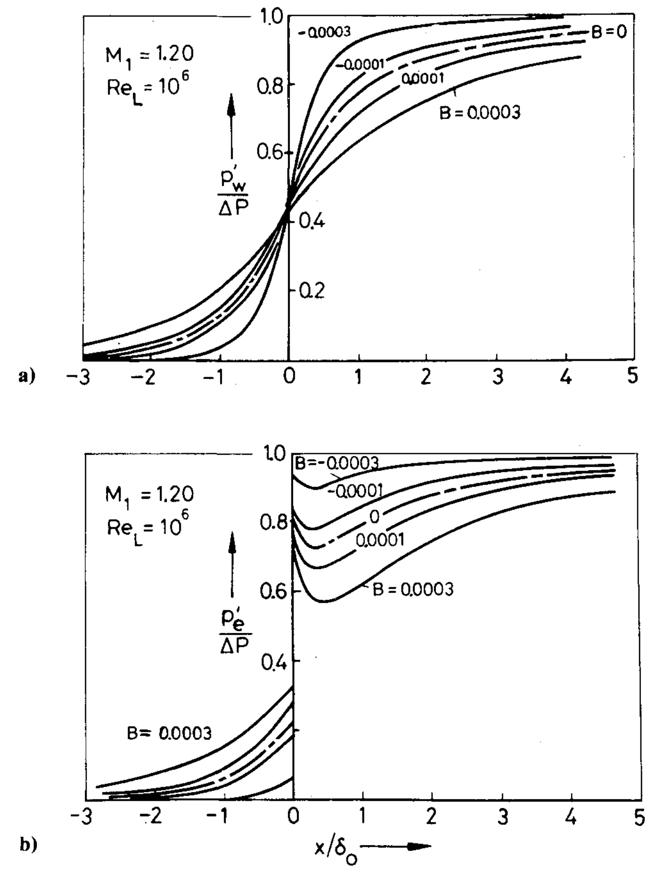

Fig. 3 Mass transfer effect study of interaction pressure along a) wall and b) boundary-layer edge.

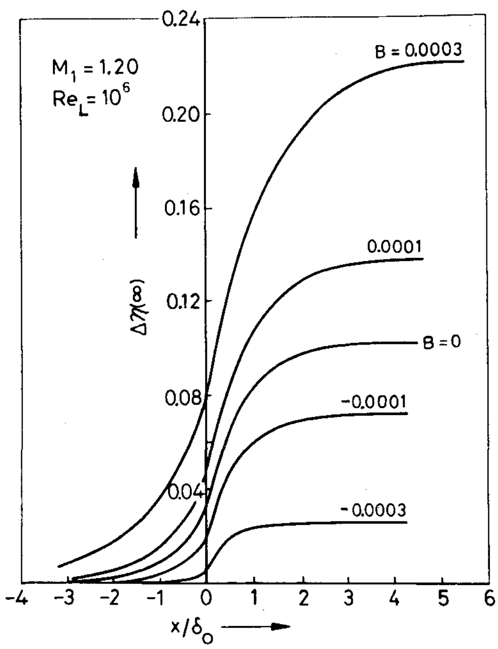

Fig. 4 Mass transfer effect on interactive thickening of the boundary layer.

The interactive local skin friction also is of interest since it bears directly on possible flow separation and/or reinitialization of boundary-layer calculations downstream of the interaction zone. As illustrated in Fig. 5, the interaction-induced adverse pressure gradient decreases $C_f(x)$ downstream toward the shock with a minimum occurring slightly

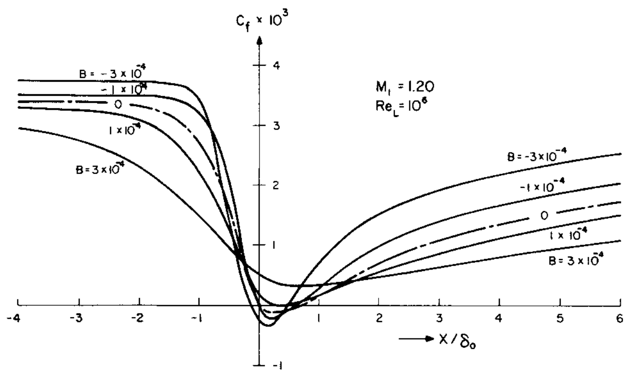

Fig. 5 Mass transfer effect on interactive skin friction.

behind it,§ followed by a subsequent slow rise far downstream. The typical influence of mass transfer is also shown in Fig. 5 and is seen to involve two opposing effects: far upstream or downstream the skin-friction-increasing effect of suction dominates (which tends to delay separation), whereas in the neighborhood of the shock foot ($|x/\delta| < 1$) the suction-induced steepening of the local adverse pressure gradient, which reduces C_f, becomes of controlling importance. Thus in contrast to what occurs in noninteracting boundary layers[29] or laminar supersonic interactions,[30] suction here actually has an adverse effect in hastening interactive incipient separation under the shock; rather, it is small amounts of *blowing* which can delay (and in the present example eliminate altogether) such separation. On the other hand, suction does exert a beneficial influence on the downstream behavior of the interaction by promotion a more rapid equilibration of the turbulent skin friction.

It is noted that while there presently exists no systematic quantitative experimental data on transonic shock-boundary-layer interactions with surface mass transfer, some observations in a channel flow test¶ have been reported[31] that qualitatively support the foregoing conclusions. Some recent experimental results on a supersonic compressive interaction flow with wall suction[32] also support our theoretically predicted reduction of interactive thickening and upstream influence. However, to the authors' knowledge, there currently exists no detailed experimental data in the shock foot region of transonic shock-unseparated turbulent boundary-layer interactions with surface mass transfer.

Concluding Remarks

This nonasymptotic study of weak normal shock/turbulent-boundary-layer interactions for two-dimensional nonseparating flows including mass transfer has shown that even small amounts of suction ($-\dot{m}_w/\rho_e u_e \lesssim 5\times10^{-4}$) appreciably reduce both the streamwise scale and thickening effect of the interaction but hasten the onset of separation slightly behind the shock foot. Equal amounts of weak blowing, on the other hand, can completely eliminate interaction-induced separation. These results were found to be mainly a consequence of the mass transfer effect on the incoming boundary-layer Mach number profile shape, which in turn significantly affects the interaction pressure distribution and hence the local skin friction. The present theory provides a useful basic analytical framework for the evaluation and parametric study of these interaction mass transfer effects in a variety of practical applications. Moreover, it provides the basis for further improvement—extension of the analytical

§Although no longer valid for separated flow where $C_f(x) < 0$ over some portion of the wall, the present skin-friction solution is still useful to indicate trends toward this situation, i.e., where and when incipient separation ($C_f \to 0$ at some x) first occurs.

¶It should be noted that the often-important blockage corrections in such tests due to interactive thickening of the wall boundary layers[19] can be significantly reduced by suction (see Fig. 4).

model to larger mass transfer rates by including their explicit effect on the viscous disturbance sublayer and mean turbulent boundary-layer eddy viscosity equations. With such added features, it should be possible to examine the basic question of how mass transfer influences incipient separation over a very wide range of suction or blowing rates as well as Mach and Reynolds numbers without the need for present-day empiricisms.

Acknowledgments

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References

[1] Pearcey, H.H., "Shock-Induced Separation and Its Prevention by Design and Boundary Layer Control," *Boundary Layer and Flow Control,* Vol. 2, edited by G. Lachman, Pergamon, New York, 1961, pp. 710-740.

[2] Bore, C.L., "On the Possibility of Deducing High Reynolds Number Characteristics Using Boundary Layer Suction," *Facilities and Techniques for Testing at Transonic Speeds,* AGARD CP-83, April 1971, pp. 23-1 to 23-9.

[3] Peake, D.J., Yoshihara, H., Zonars, D., and Carter, W., "The Transonic Performance of Two-Dimensional Jet-Flapped Airfoils at High Reynolds Numbers," *Facilities and Techniques for Testing at Transonic Speeds,* AGARD CP-83, April 1971, pp. 7-1 to 7-39.

[4] Green, J.E., "Interactions Between Shock Waves and Boundary Layers," *Progress of Aeronautical Science,* Vol. 11, Pergamon, New York, 1965, p. 319.

[5] Piva, R. and de Jesus, C., "The Effect of Injection on Base Drag at Transonic Speed," New York University, Department of Aeronautics, Rept. AA-12-19 (ASOSR TR 72-2044), July 1972.

[6] Kruse, H., "Boundary Layer Calculation of an Effusion-Cooled Turbine Blade," AIAA Paper 78-90, Huntsville, Ala., 1978.

[7] Schlichting, M., *Boundary Layer Theory,* 6th Ed., McGraw-Hill, New York, 1974, p. 644.

[8] Chang, P.K., *Control of Flow Separation,* McGraw-Hill, New York, 1976, pp. 298-351.

[9] Melnik, R.E. and Grossman, B., "Analysis of the Interaction of a Weak Normal Shock Wave with a Turbulent Boundary Layer," AIAA Paper 74-598, Palo Alto, Calif., June 1974.

[10] Adamson, T.C. and Feo, A., "Interaction Between a Shock Wave and a Turbulent Layer in Transonic Flow," *SIAM Journal of Applied Mathematics,* Vol. 29, July 1975, pp. 121-141.

[11] Inger, G.R. and Mason, W.H., "Analytical Theory of Transonic Normal Shock-Boundary Layer Interaction," *AIAA Journal,* Vol. 14, Sept. 1976, pp. 1266-1272.

[12] Stratford, B.S., "The Prediction of Separation of the Turbulent Boundary Layer," *Journal of Fluid Mechanics,* Vol. 5, 1959, pp. 1-16.

[13] Townsend, A., "The Behavior of a Turbulent Boundary Layer Near Separation," *Journal of Fluid Mechanics,* Vol. 12, 1967, pp. 536-554.

[14] Inger, G.R., "Upstream Influence in Interacting Non-separated Turbulent Boundary Layers," presented at the Workshop on Viscous Interaction and Boundary Layer Separation, Ohio State University, Columbus, Ohio, Aug. 17, 1976 (VPI & SU report in preparation).

[15] Inger, G.R., "Shock Penetration and Lateral Pressure Gradient Effects on Transonic Viscous Interactions," *AIAA Journal,* Vol. 15, Aug. 1977, pp. 1198-1200.

[16] Mason, W.H. and Inger, G.R., "Analytical Study of Transonic Normal Shock-Boundary Layer Interaction," AIAA Paper 75-831, Hartford, Conn., June 1975.

[17] Lighthill, M.J., "On Boundary Layers and Upstream Influence; II. Supersonic Flow Without Separation," *Proceedings of the Royal Society, Series A,* Vol. 217, San Diego, Calif., 1953, pp. 478-507.

[18] Stewartson, K. and Williams, P.G., "Self-Induced Separation," *Proceedings of the Royal Society, Series A,* Vol. 312, San Diego, Calif., 1969, pp. 181-206.

[19] Inger, G.R., "Analysis of Transonic Normal Shock-Boundary Layer Interaction and Comparisons with Experiment," AIAA Paper 76-331, San Diego, Calif., July 1976; VPI&SU Rept. Aero-053, Blacksburg, Va., Aug. 1976.

[20] Inger, G.R. and Zee, S., "Transonic Shock-Turbulent Boundary-Layer Interaction with Suction or Blowing," VPI&SU Rept. Aero-073, Blacksburg, Va., Sept. 1977.

[21] Chen, T.S., Sparrow, E.M., and Tsou, F.K., "The Effect of Main-flow Transverse Velocities in Linear Stability Theory," *Journal of Fluid Mechanics,* Vol. 50, San Diego, Calif., 1971, pp. 741-750.

[22] Inger, G.R. and Williams, E.P., "Subsonic and Supersonic Boundary-Layer Flow Past a Wavy Wall," *AIAA Journal,* Vol. 10, May 1972, pp. 636-642.

[23] Dorrance, W.H., *Viscous Hypersonic Flow,* McGraw-Hill, New York, 1962, pp. 218-219.

[24] Conrad, P., Donaldson, C. du P., and Snedeker, R., "A Study of the Model Response Approach to Patterned Ablation Including Experiment Definition," Aeronautical Research of Princeton, Inc., Princeton, N.J., SAMSO TR-70-213, 1969.

[25] Kutateladze, S.S. and Leont'av, A.I., *Turbulent Boundary Layers in Compressible Gases,* Edward Arnold, London, 1964, p. 24.

[26] Coles, D., "A Survey of Turbulent Boundary Layers with Mass Transfer," RAND Corp. Rept. P4697, Sept. 1971.

[27] Schetz, J.A. and Nerney, B., "Turbulent Boundary Layer with Injection and Roughness," *AIAA Journal,* Vol. 15, Sept. 1977, pp. 1288-1294.

[28] Panaras, A.G. and Inger, G.R., "Normal Shock-Boundary Layer Interaction in Transonic Speed in the Presence of Streamwise Pressure Gradient," ASME Paper 77-GT-34, International Gas Turbine Conference, Phila., Pa., April 1977.

[29] Wuest, W., "Theory of Boundary Layer Suction to Prevent Separation," *Boundary Layer and Flow Control,* Vol. 2, edited by G. Lachman, Pergamon, New York, 1961, pp. 196-250.

[30] Pate, S.R., "Experiments on Control of Supersonic Transitional Flow Separation Using Distributed Suction," *AIAA Journal,* Vol. 7, May 1969, pp. 847-851.

[31] LeBlanc, R. and Göthals, R., "Etude de Phenomenes d'Interaction Onde de Choc Normale—Couche Limit Turbulence," Centre d'Etudes Aerodynamiques et Thermiques, University of Poitiers, Poitiers, France, Oct. 1974.

[32] Gai, S.L., "Shock-Wave Boundary Layer Interaction with Suction," *Zeitschrift für Flugwissenschaften und Weltraumforschung Band I,* Vol. 2, March-April 1977.